1906
P84

POST

Coefficients of Discharge for Small Weirs

Civil Engineering

B S

1906

UNIVERSITY
OF
ILL. LIBRARY

UNIVERSITY OF ILLINOIS
LIBRARY

Class 1906
Book P84
Volume

Je 06-10M

181/67 Inuf D

COEFFICIENTS OF DISCHARGE
FOR SMALL WEIRS

BY

ROYAL ELMER POST

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

PRESENTED JUNE 1906

88066

1906
P84

UNIVERSITY OF ILLINOIS

May 29, 1906

This is to certify that the following thesis prepared under the direction of Professor A. N. Talbot, Head of the Department of Theoretical and Applied Mechanics, by

ROYAL ELMER POST

entitled COEFFICIENTS OF DISCHARGE FOR SMALL WEIRS

is hereby approved by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

Ira O. Baker

Head of Department of Civil Engineering

95
31 Ag '06

The Rare Book & Manuscript Library UIUC

INTRODUCTION

The measurement of water in open channels is most easily and accurately made by means of weirs. When the quantity of water to be measured is large, weir measurement is commonly used and the coefficients for large weirs have been determined repeatedly, so that the values are probably more accurately known than are some of the other factors entering into the problem. Very often, however, in connection with waterworks, irrigation, and power development, smaller quantities of water flowing in open channels have to be measured. This measurement may be made cheaply and conveniently by means of small, fully contracted weirs. The coefficients for small weirs have not been determined as fully or as accurately as have those for large weirs. This is especially true for low heads, and while it is probable that the coefficients for heads over 0.5 ft. are more often used than those for lower heads it is desirable to get the coefficients for low heads because of what they add to the general knowledge of weirs, and to the limitations of the accurate measurement of water

by weirs.

The following thesis is an effort to determine the value, accuracy, and variation of the coefficient with low heads. A complete calibration of the several weirs will not be attempted.

THE STANDARD WEIR FORMULA

The theoretical formula for weirs is derived from that for orifices by considering the head on the top of the orifice zero.

The formula for orifices is:-

$$Q = \frac{2}{3} b \sqrt{2g} \left(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}} \right)$$

where. Q = discharge is cu. ft. per sec.,
b = breadth of orifice in ft.,
h_1 = head on top of orifice in ft.,
and h_2 = head on bottom of orifice in ft.

Making $h_1 = 0$, and calling b for the orifice L for the weir we have:-

$$Q = \frac{2}{3} L \sqrt{2g} H^{\frac{3}{2}}$$

in which H is the head on the crest measured at some point back of the surface curve, since there is no surface curve in the case of orifices and the modified formula does not take into account that in the case of weirs.

This formula gives the theoretical discharge when there is no velocity at the place where H is measured i.e. when the area of the weir opening is small compared to the area of the stream cross-section. The change in the formula made by velocity of approach will not be considered as there was practically none in the experiments made.

The actual discharge is always less than the theoretical discharge, due to the contraction of the stream, the friction of the air, and the resistance of the edges of the weir. Various ways of modifying the theoretical formula to take this into account have been proposed, but it is generally conceded that the form in which the theoretical formula is modified by a single experimental coefficient c is best. The practical formula is then-

$$Q = c\,\frac{2}{3}\sqrt{2g}\,LH^{\frac{3}{2}}$$

where c is an experimental coefficient depending upon the length of the weir, the condition of the edges, the amount of contraction, and the head of water flowing over the crest. For a given place g is constant so that the factors $\frac{2}{3}\sqrt{2g}$ may be combined into a single constant whose value for $\sqrt{2g} = 8.02$ is 5.347. For any particular weir the factors 5.347 and

L may be combined into a single constant K. The formula then becomes —

$$Q = cKH^{\frac{3}{2}} \text{ and } c = \frac{Q}{KH^{\frac{3}{2}}}.$$

This form is the most convenient for the calculation of c.

For purposes of tabulation the form of the formula $Q = c\frac{2}{3}\sqrt{2g}\,LH^{\frac{3}{2}}$ is usually taken as a base and the values of c for different lengths and heads tabulated. In the recent government publication, Water Supply and Irrigation Paper No 150, entitled Weir Experiments Coefficients and Formulas, the standard formula is modified to the form $Q = CLH^{\frac{3}{2}}$, where C is found directly by dividing the Q obtained in the experiment by the three-halves power of H times L. It may be deduced from the c of the other form of tabulation by multiplying c by $\frac{2}{3}\sqrt{2g}$, or 5.347, if $\sqrt{2g} = 8.02$. The form $Q = CLH^{\frac{3}{2}}$ seems much the better for ease and rapidity of computation of Q and in such work as the weir measurement of water, where the accuracy can in no case be very great it seems useless to take into account the small variation of g from place to place. Values of C as well as c are tabulated in the results of the experiments of this thesis.

DISCUSSION OF FORMER EXPERIMENTS ON SHORT RECTANGULAR FULLY CONTRACTED WEIRS

A great many experiments have been made on long weirs of all kinds and many on short suppressed weirs, but there seems to be little experimental data on rectangular, sharp-crested, fully contracted weirs, less than three feet long. In the comparatively few experiments which have been made low heads received little attention, - most of the experiments being with heads from 0.5 ft. upwards. Following is a short discussion and tables of results of the several series of experiments which have been made on short, contracted weirs. They are, so far as is known the only ones which have been published.

Experiments of Poncelet and Lesbros

In 1828 Poncelet and Lesbros made some experiments on a weir .66 ft long with complete end contractions and very nearly complete bottom contraction - the depth of water below the crest of the weir being 1.77 ft. The results of these experiments are given in the table following.

PONCELET AND LESBROS

Full contraction. Discharge free into air.
H measured 11ft. above weir.

No	H	C	Remarks
35	.6821	.583	L = .6562 ft.
36	.5351	.589	
37	.3376	.591	
38	.1985	.600	
39	.1463	.608	
40	.0771	.622	

Experiments of Lesbros

In 1834 Lesbros obtained the following results on a weir .66 ft. long. He used various forms of approach giving partial contraction for high heads and full contraction for low heads. The contraction was in all cases nearly full

LESBROS

Nearly full contraction. Discharge free into air.
H measured 11ft above weir.

No	H	C	Remarks
41	.5955	.590	L = .6562 ft.
42	.3625	.596	
43	.1788	.610	
44	.0958	.628	
45	.3724	.596	
46	.1070	.630	

Experiments of Fteley and Stearns

Fteley and Stearns made a number of experiments on a weir three feet long with more or less end contraction, but since the quantities of water were determined indirectly by measurement with a five foot weir, and since the contraction was in no case full and in many of the experiments far from full, these experiments will not be discussed.

Experiments of Hamilton Smith

From 1874 to 1876 Hamilton Smith made a series of experiments on weirs and orifices at Columbia Hill, Cal., for three mining companies that bought water measured by the crude methods in use at the time and wished to know the exact discharge of the weirs and orifices they were using.

His apparatus consisted of a standard weir fed by a canal leading from a reservoir, which gave a head nearly constant, and a wooden measuring tank of thirteen hundred cubic feet capacity. He measured the heads on the weir by a hook gauge placed 7 ft above the weir. The time recorded was that required for the tank to fill from a point fixed near the bottom of the tank to one fixed near the top. Allowance was made for leakage spillage etc. and every precaution

taken to get accurate results. No velocity of approach was considered. The weir itself was rectangular, of iron plate 2.587 ft. long, with thin sharp edges and square corners. It was set in the end of the feeding canal which had sufficient area of cross-section to insure full contraction of the escaping stream.

In general the experiments were made with the greatest possible care by an experienced observer and the results are undoubtedly very accurate. It will be seen, however, by an examination of the table of results below that the lowest "head used" was 0.5659 ft. The results therefore determine no points on the weir curve for heads below 0.5659 ft.

HAMILTON SMITH

Nearly full contraction. Discharge free into air.

$L = 2.586$ ft.

No	H	C	No	H	C
57	.5659	.6087	63	1.1063	.5878
58	.6163	.6032	64	1.2033	.5910
59	.6470	.6030	65	1.3257	.5804
60	.6703	.6020	66	1.5391	.5918
61	.7072	.6021	67	1.7195	.5840
62	1.0681	.5890	68	1.7327	.5812

COEFFICIENTS COMMONLY QUOTED

The tabulated values of "c" found in Merriman and other texts are those deduced by Smith from curves which he plotted in connection with all the experimental data available in 1880, when his book on hydraulics was written. His curves form a graphical theory of the variation of "c" as indicated by experimental data.

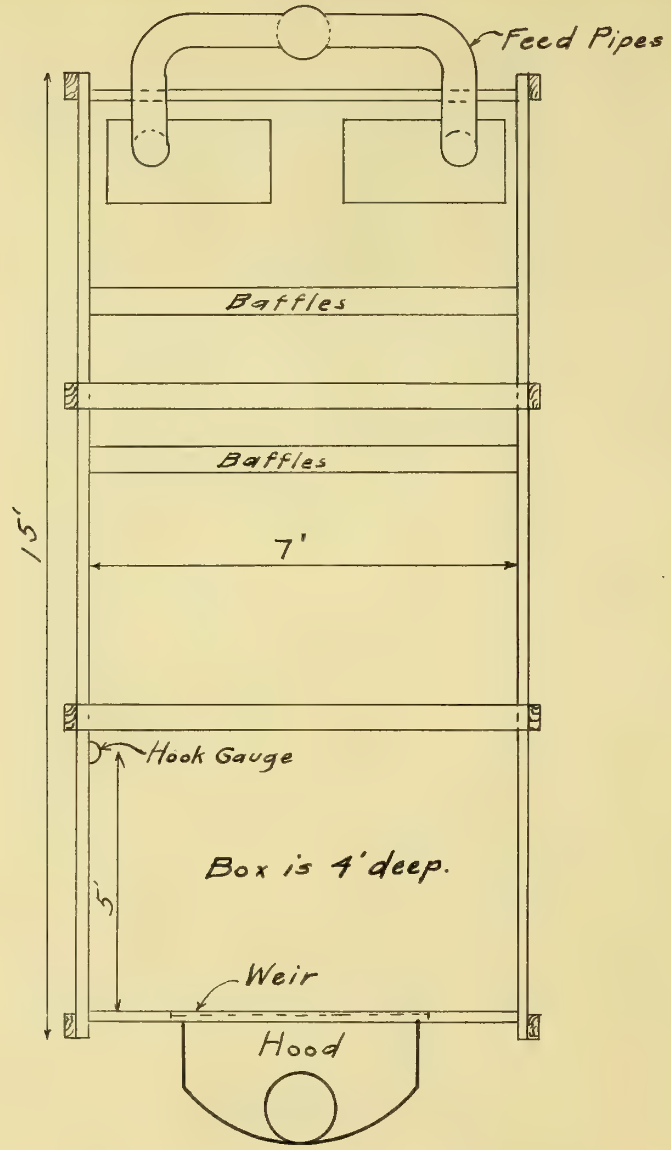
The method of arriving at the results expressed by Smith's curves will be given briefly. ~~Set~~ $c_c = c$ for contracted weirs, $c_s = c$ for suppressed weirs, and $c_i = c$ for a weir of infinite length. Obviously the longer the weir the nearer alike are c_c and c_s, and for a weir of infinite length $c_c = c_s = c_i$. From a consideration of the experimental values of c. Smith drew three conclusions:- (1) that all values of c_s are larger than the values of c_c for the same heads; (2) that c_s increases as L diminishes; (3) that c_c diminishes as L diminishes. He then

stated the proposition:—"that, compared with the curve of c for a weir of infinite length, with full contraction c_c diminishes as L diminishes and with contraction suppressed c_s increases as L diminishes." He then constructed the curves of c_i in what he considered its correct position between the curves of c_c and c_s for the longest weirs he had results on. He plotted the values of c as ordinates and those of H as abscissae. Then according to the proposition stated above as L diminishes the curves of c_s become higher and higher and those of c_c lower and lower. The probable location of the curves for each length of weir he determined from the plotted experiments or else interpolated between the curves which were determined more or less closely by experiments.

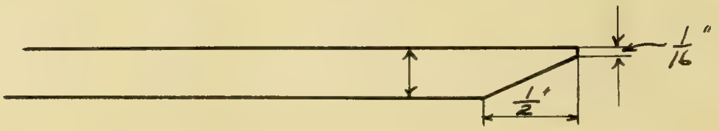
As has been stated Smith had the results of many experiments on suppressed weirs and on long "contracted weirs" so that for these his curves are pretty well determined by experimental data. In the curves of c_c for 1 ft, 2 ft. and 3 ft. weirs the results given in the preceding discussion were the only ones used. The curves for the 1 ft and 2 ft weirs were interpolated

from results on weirs .66 ft long and 2.0 ft long. The points on the curve for the 3 ft weir are Fteley and Stearns values of c determined indirectly by comparison with a five foot weir.

While the theory of these curves seems to be correct and while the curves themselves are an excellent interpretation of the data at hand, there are hardly enough points on the short weir curves for high heads and certainly not nearly enough for low heads to fix beyond question the values of c as taken from them. Some of the points on the curves are fixed by a single experiment and the experience of the writer on similar weirs leads him to believe that this cannot be safely done. The most careful work in this line is liable to considerable error and an average of representative results seems the only way to fix accurately the value of c for any particular length and head. The curves represent a painstaking and comprehensive analysis of the data Smith could get at the time, but it is desirable to get more experimental results, on short weirs especially, before finally accepting his values as the true values of c for standard thin-edged weirs.



PLAN OF WEIR BOX.



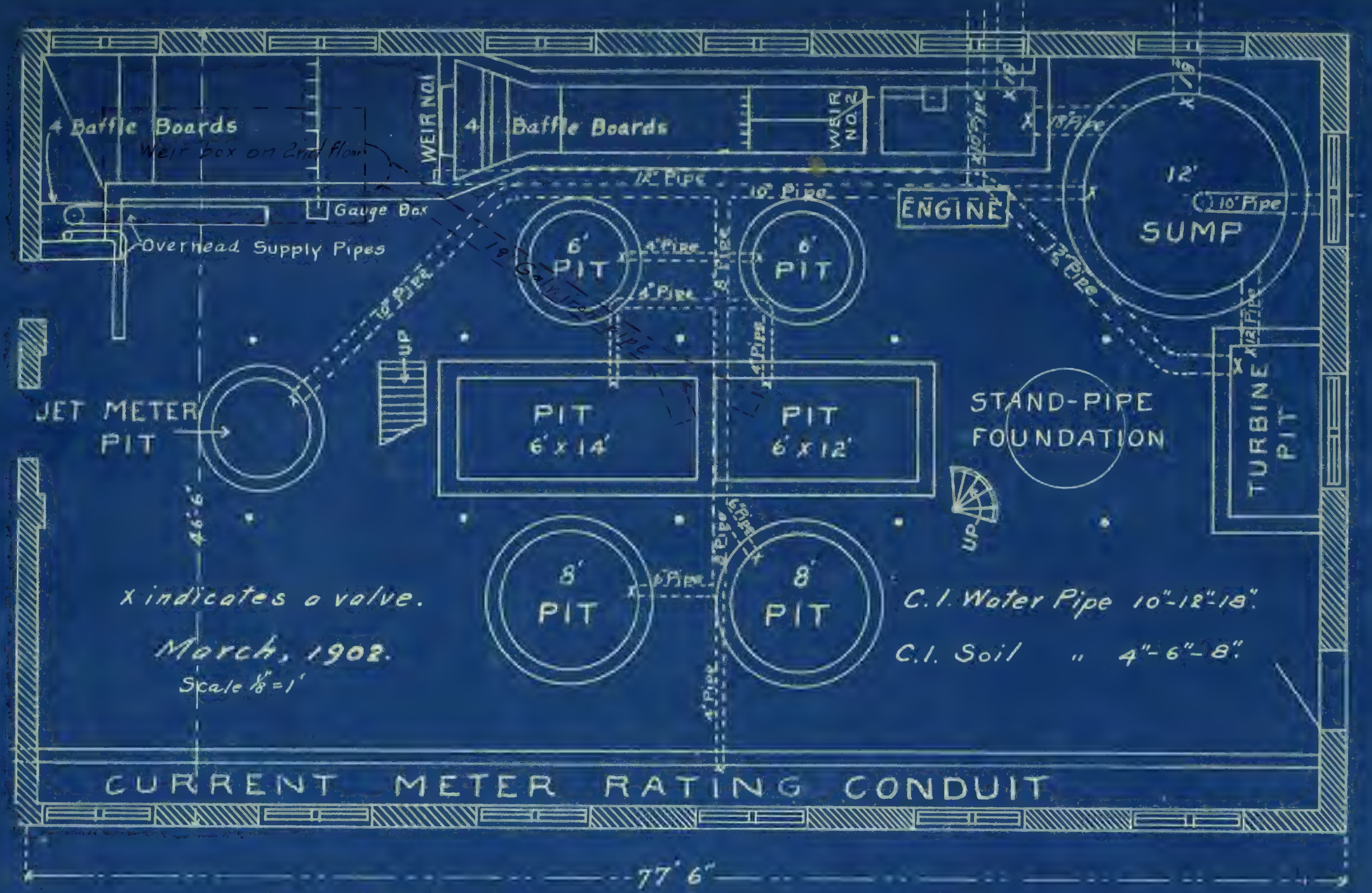
CROSS-SECTION OF WEIR EDGE.

DESCRIPTION OF APPARATUS

The apparatus consists in general of a large weir box fed by an 8 in. main situated on the second floor of the Hydraulic Laboratory and two concrete measuring pits situated on the first floor. There is an 18" galvanized iron pipe leading from the hood into which the weir discharges to the measuring pits. In the end of this pipe is a Y which divides the flow proportionately between the two pits. Sacks were tied on the ends of this Y to prevent splashing and spillage. The pits are connected at the bottom by a 4" pipe which helps to keep the water level in the two pits the same. The dimensions and general design of the weir box may be seen from the accompanying sketch.

The weirs themselves are cut out of steel plates which fit into the end of the weir box. These plates are of the same exterior dimensions and are set flush with the end of the weir box and fastened by countersunk bolts. The thickness of plate, thickness of edge and bevel of edge may be seen from the sketch on opposite page.

The hook gauge, which is fixed to the weir



x indicates a valve.

March, 1902.

Scale 1/8" = 1'

C. I. Water Pipe 10"-12"-18".

C. I. Soil " 4"-6"-8".

BASEMENT PLAN OF HYDRAULIC LABORATORY
SHOWING LOCATION OF UNDERGROUND PIPES AND VALVES

top 5 ft. back of the weir, is a Gurley gage reading to 0.001 ft. The pits are each fitted with an 8 in. galvanized iron pipe fastened in a vertical position to lugs set in the concrete and extending nearly to the bottom of the pit. This pipe is for the purpose of getting still water in which to measure depths. On the inside of each pipe are two fixed hooks one near the bottom and one near the top of the pits. The use of these will be explained later. A stop watch was used to take the time.

For an overflow to the standpipe one of the pipes leading from the standpipe about 25 ft. from its base was used. This pipe was isolated by shutting the proper valves and connected with the sump.

The general layout of the laboratory may be seen from the accompanying blue print.

METHOD OF EXPERIMENTATION

From a consideration of the methods used by Hamilton Smith and others two things were seen to be most important, a constant or nearly constant head of water on the weir and some way of accurately determining Q, the flow in cu. ft. per sec.

The overflow to the stand pipe insured a head very nearly constant in every case — the maximum variation being 0.01 ft. and the usual variation not more than 0.001 ft. Often the readings were exactly the same throughout the experiment.

In order to determine Q the pipes were measured with a wooden measuring rod at three places - top bottom and middle and the average area calculated. This area multiplied by the distance between the hooks inside the galvanized iron pipe gave the total volume of water between the hooks or in other words the total flow over the weir during the time of one experiment as recorded by the stop-watch. This total volume divided by the time in seconds gave Q in cu. ft. per sec.

It is thought that the above method in which a constant volume of water is used and the time varied is much simpler and more likely to give accurate results than a method involving the use of a shifting discharge pipe. In the latter method the pit has to be calibrated for all depths instead of one and there is always a chance for bungling in giving and obeying signals to shift the discharge pipe.

The zero of the hook gauge was obtained by adjusting the water level in the box so that the edge of a white card appeared to just meet its image in the water at the same instant the card touched the crest of the weir, and then taking the reading of the gauge. The zero reading was determined four to six times in each series and the value nearest the mean choosen as the zero of the hook gauge. The values so choosen are thought to be within at least 0.0005 ft. of the true H_0.

The method of procedure was as follows. The valve of the inlet pipe was opened until the desired flow was obtained. The capacity of the pits below the lower hook was sufficient to insure a uniform flow before the experiment started. When the level of the water in the pits reached the lower hook the stop

watch was snapped and the hook gauge read from then until the end of the experiment at intervals of one-half second or one second, depending on the steadiness of the head. When the water level reached the top of hook the watch was snapped again, the water shut off, and the pits drained for the next experiment. The usual rate of running after getting started was from two to three experiments per hour.

The greatest chance for error in these experiments was caused by the occasional differing in level of the water in the two pits near the end of an experiment. The 4 in. pipe connecting the pits proved too small to take care of any large differences of flow between the two branches of the Y, and so the water level in the two pits would differ more or less whenever the Y was not set at nearly the right angle. This occured only under high heads as under low heads the Y could readily be adjusted nearly enough right to insure the 4 in. pipe taking care of the small difference of flow between the two branches. The error was approximately corrected by measuring the difference in level of the two pits and allowing the water in the one used in getting the time to be half the difference in level above or below the hook when the watch was snapped.

DISCUSSION OF RESULTS

The separate weir curves pp22,24,&26 represent the most probable values of the coefficient as indicated by the series of experiments on the weir in question without considering the values determined in the other two series. The curves for the several weirs plotted on a single sheet, p 27, represent the probable values of the coefficient as indicated by the experiments on the three weirs considered in reference to each other, and according to Smith's proposition that for contracted weirs the longer the weir the larger the coefficient for the same head. These curves may be used for determining "c" until more extensive experiments have been made on short weirs for low heads.

The results of this thesis give uniformly higher coefficients for the range of heads used in the experiments than are derived by Hamilton Smith. The writer feels confident that the large differences between the values found and those derived

by Smith is not due to any constant error in the experiments because the experiments were made in such a way that errors of any importance in H_o or Q would certainly have been detected. The only other explanation of the wide differences in value between the results of this thesis and Smith's values is that Smith's values are not reliable for low heads. While the results of the forgoing experiments are not extensive enough to fully establish new values of c, they show that Smiths values are considerably too low for contracted weirs of 2 ft. and 3 ft. lengths with heads below 0.4 ft., and give new values of c which may be used until better ones are determined.

The results also show very clearly the larger errors incident to measuring water by weirs when the head is low. Not only do slight errors in H_o and H make large errors in Q, but there is no certainty of the value of c within 2% or 3% and due to some peculiarity of the weir or approach c might be 5% in error. This would give a possible error of 7% or 8% and the probable error would also be high. The text books emphasize the undesirability of low heads in weir measurement of water, but according

to the results of this thesis the errors of observation and inaccuracy of the coefficient are so large and variable as to prohibit the use of low heads when any degree of accuracy is desired.

RESULTS

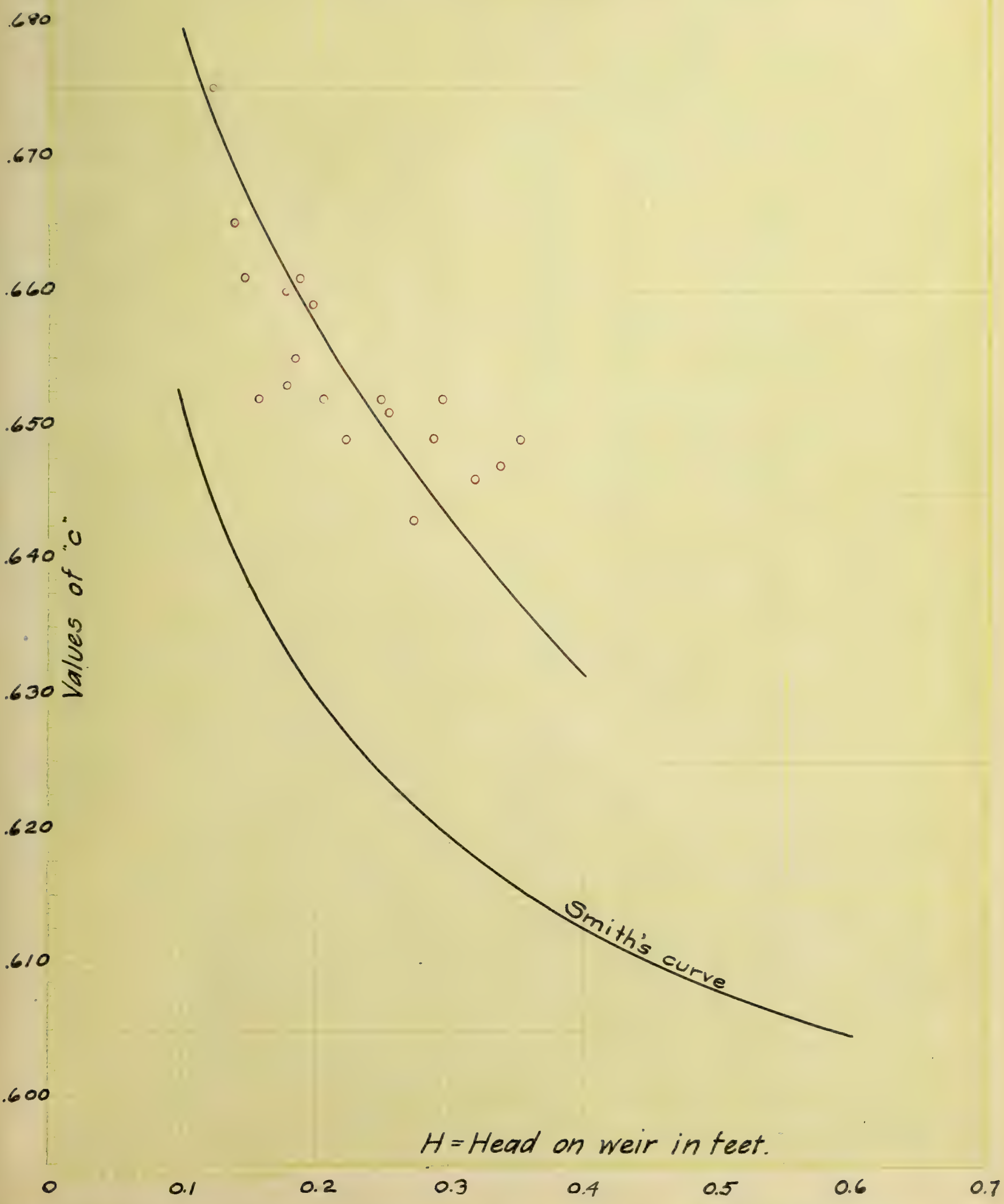
3 ft Weir. Full Contraction. $Q = 648.72$ cu.ft.

c for formula $Q = \frac{2}{3} c L H^{\frac{3}{2}} \sqrt{2g}$

C " " $Q = C L H^{\frac{3}{2}}$

No	H	T		c	C
1	.123	23'	07"	.675	3.609
2	.138	19'	44"	.665	3.556
3	.147	18'	05"	.661	3.534
4	.156	16'	47"	.652	3486
5	.178	13'	35"	.660	3.529
6	.179	13'	36"	.653	3.492
7	.184	13'	02"	.655	3.502
8	.188	12'	31"	.661	3.534
9	.198	11'	37"	.659	3.524
10	.205	11'	09"	.652	3.486
11	.222	9'	52"	.649	3.470
12	.248	8'	23"	.652	3.486
13	.253	8'	07"	.651	3.481
14	.272	7'	23"	.643	3.438
15	.287	6'	45"	.649	3470
16	.294	6'	29"	.652	3.486
17	.318	5'	49"	.646	3454
18	.336	5'	21"	.647	3459
19	.353	4'	57"	.649	3470

CURVE FOR 3ft WEIR



RESULTS

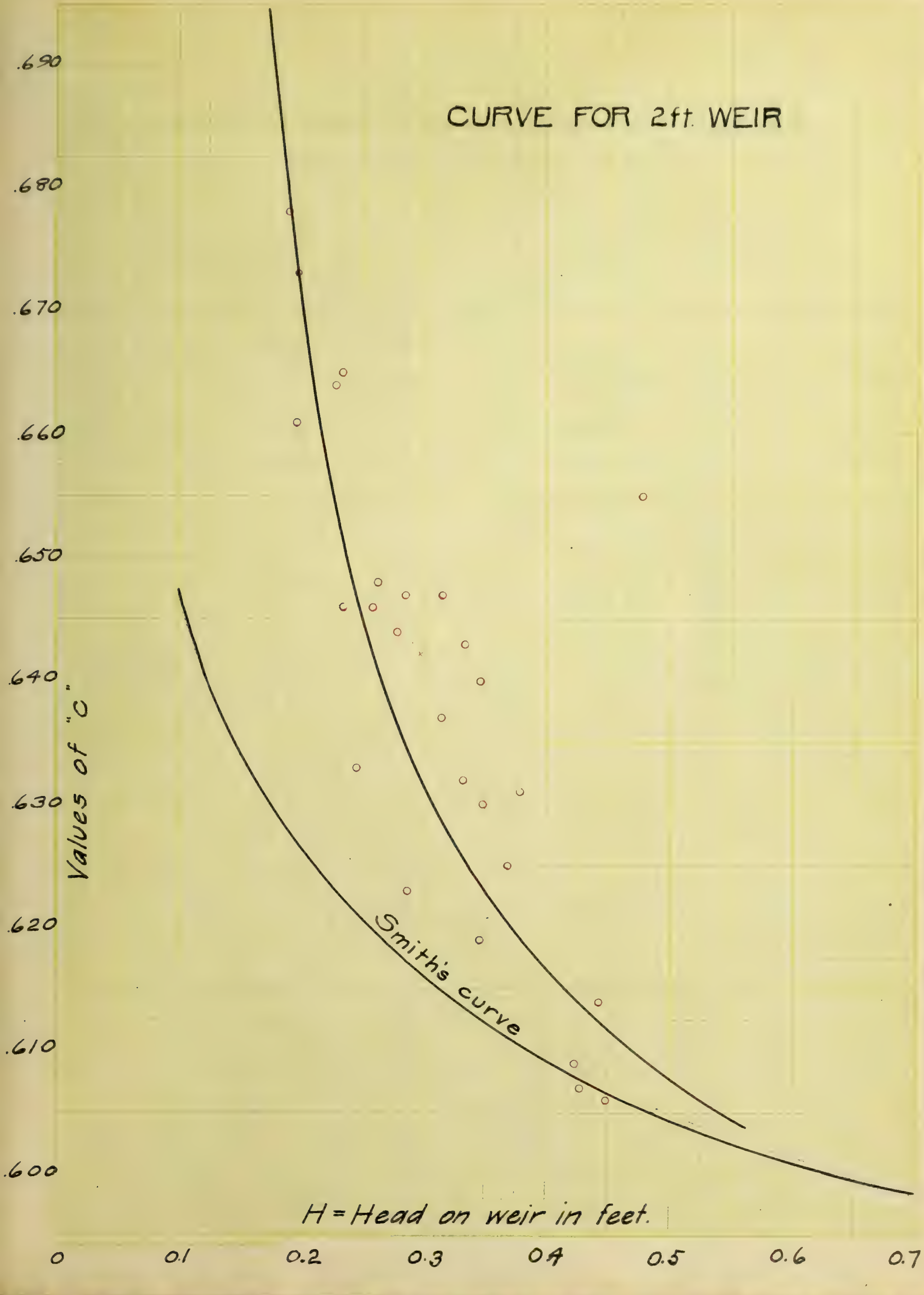
2 ft Weir. Full Contraction. Q=643.92

c for formula $Q=\frac{2}{3}cLH^{\frac{3}{2}}\sqrt{2g}$

C " " $Q=CLH^{\frac{3}{2}}$

No	H	T	c	C	
1	.134	28' 32"	.697	3.727	
2	.159	22' 06"	.697	3.727	
3	.190	17' 24"	678	3.625	
4	.196	17' 30"	.661	3.534	
5	.198	16' 29"	.673	3.598	
6	.229	13' 26"	.664	3.550	
7	.234	12' 58"	.665	3.556	
8	*.234	13' 43"	.646	3.454	
9	.244	12' 21"	633	3.384	
10	.258	11' 50"	.646	3454	
11	.262	11' 15"	.648	3465	
12	.278	10' 21"	644	3.443	
13	.285	9' 55"	.647	3.459	
14	*.286	10' 32"	.623		
15	.314	8' 43"	.637	3.406	
16	.315	8' 40"	.647	3.459	
17	*.318	8' 58"	595		
18	.332	8' 05"	.632	3.379	
19	.333	7' 54"	.643	3.438	
20	.346	7' 30"	.640	3422	
21	*.346	7' 58"	.619		
22	.349	7' 43"	.630	3.369	
23	.369	7' 10"	.625	3.342	
24	.379	6' 49"	.631	3.374	
25	.424	5' 58"	.609	3.256	
26	.428	4' 47"	.607	3246	Q=521.58
27	*.443	5' 24"	.614	3.283	
28	448	5' 31"	.606	3.240	
29	*477	4' 32"	655		

Results marked * inaccurate due to spillage etc.

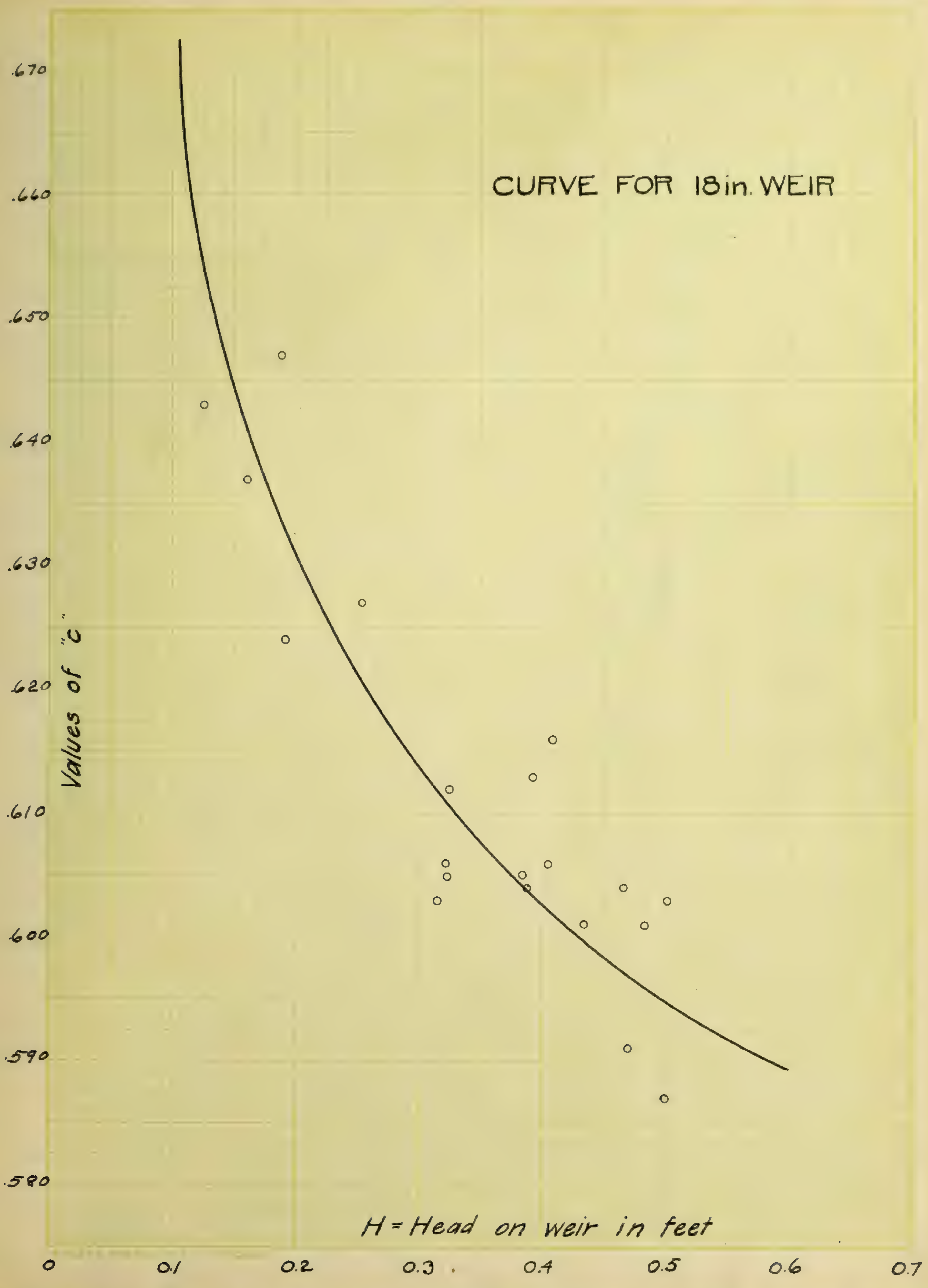

CURVE FOR 2ft. WEIR

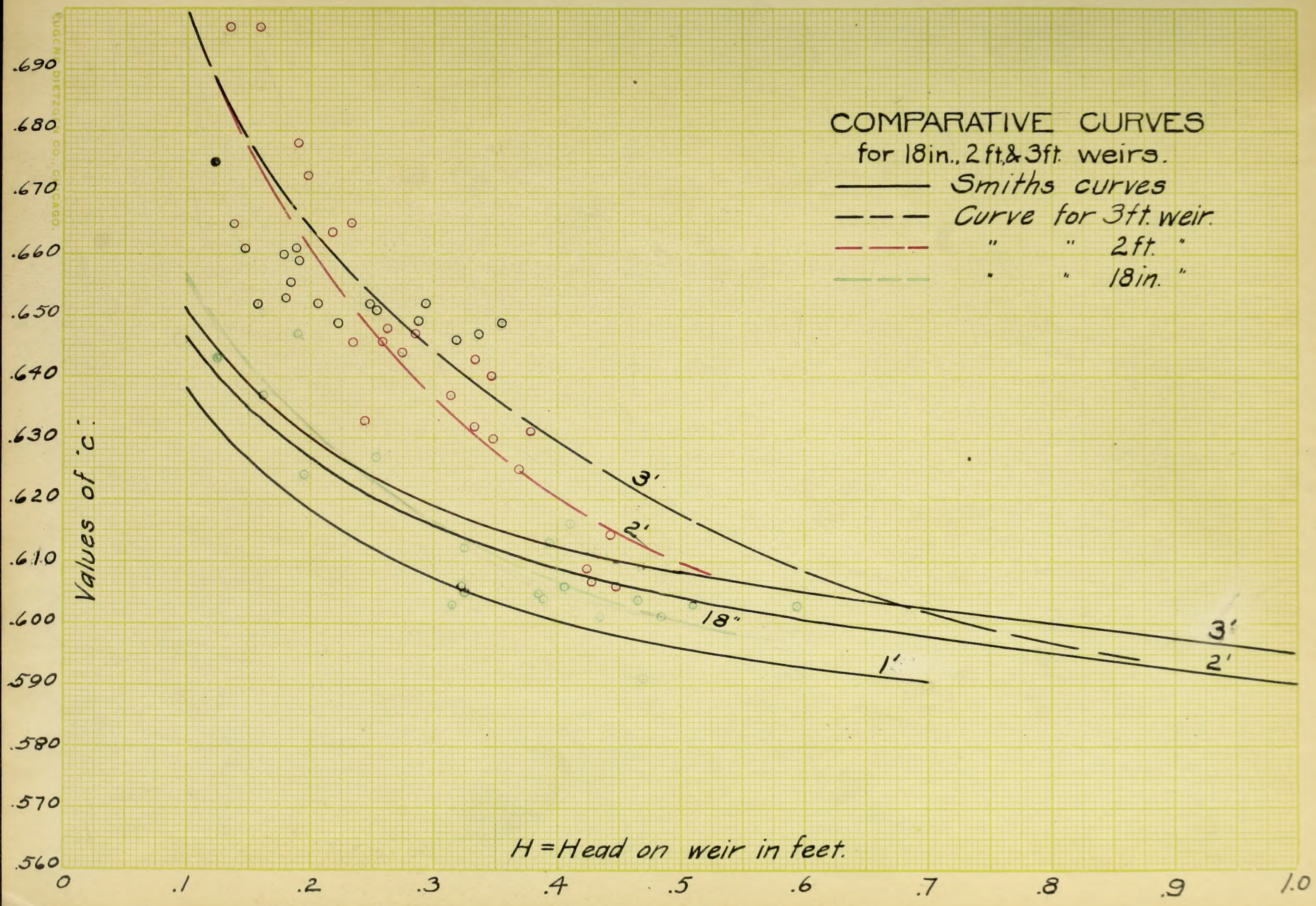
RESULTS

18 inch Weir. Full Contraction. Q=645.66

c for formula $Q=\frac{2}{3}cLH^{\frac{3}{2}}\sqrt{2g}$

No	H	T		c	C
1	.315	12′	35″	.603	C will not be given for this weir.
2	.323	12′	04″	.606	
3	.324	12′	02″	.605	
4	.385	9′	17″	.605	
5	.389	9′	09″	.604	
6	.393	8′	53″	.613	
7	.405	8′	35″	.606	
8	.410	8′	18″	.616	
9	.435	7′	47″	.601	
10	.465	7′	00″	.604	
11	.470	7′	03″	.591	
12	.483	6′	39″	.601	
13	.501	6′	28″	.603	
14	.597	4′	49″	.603	
15	.124	25′	03″	.643	
16	.161	17′	07″	.637	
17	.189	13′	34″	.647	
18	.194	13′	05″	.624	
19	.253	8′	47″	.627	
20	.325	6′	14″	.612	


CURVE FOR 18in. WEIR
.670
.660
.650
.640
.630
.620
.610
.600
.590
.580
Values of "c"
H = Head on weir in feet
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


COMPARATIVE CURVES
for 18in., 2 ft.& 3ft. weirs.
Smiths curves
Curve for 3ft. weir.
" " 2ft. "
" " 18in. "
3'
2'
18"
3'
2'
1'
Values of 'c'.
H = Head on weir in feet.
.690
.680
.670
.660
.650
.640
.630
.620
.610
.600
.590
.580
.570
.560
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
EUGENE DIETZGEN CO., CHICAGO.